

ITEP-TH-07/97
NBI-HE-97-01
hep-th/9701151
January, 1997

Properties of D-Branes in Matrix Model of IIB Superstring

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Abstract

We discuss properties of D-brane configurations in the matrix model of type IIB superstring recently proposed by Ishibashi, Kawai, Kitazawa and Tsuchiya. We calculate central charges in supersymmetry algebra at infinite N and associate them with one- and five-branes present in IIB superstring theory. We consider classical solutions associated with static three- and five-branes and calculate their interactions at one loop in the matrix model. We discuss some aspects of the matrix-model formulation of IIB superstring.

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1 Introduction

It has been recently proposed by Banks, Fischler, Shenker and Susskind [1] that nonperturbative dynamics of M theory is described by a supersymmetric $N \times N$ matrix quantum mechanics in the limit of large N . This Matrix theory naturally includes Witten's description [2] of bound states of D(irichlet)-branes by matrices and is shown [1, 3, 4, 5, 6, 7] to correctly reproduce properties of Dp-branes with even p ($p = 0, 2, 4, \dots$) incorporated by type IIA superstring theory.

Another matrix model which is an analogue of the BFSS matrix model [1] for type IIB superstring has been proposed by Ishibashi, Kawai, Kitazawa and Tsuchiya [8]. This model is defined by the vacuum amplitude

$$Z = \sum_{n=1}^{\infty} \int dA d\psi e^{iS} \quad (1.1)$$

with the action being

$$S = \frac{1}{g_s(\alpha')^2} \left(\frac{1}{4} \text{Tr} [A_\mu, A_\nu]^2 + \frac{1}{2} \text{Tr} (\bar{\psi} \Gamma^\mu [A_\mu, \psi]) \right) + \beta n. \quad (1.2)$$

Here A_μ^{ij} and ψ_α^{ij} are $n \times n$ Hermitian bosonic and fermionic matrices, respectively. The vector index μ runs from 0 to 9 and the spinor index α runs from 1 to 32. The fermion ψ is a Majorana–Weyl spinor which satisfies the condition $\Gamma_{11}\psi = \psi$. The summation over μ is understood with ten-dimensional Minkowski metric. We prefer to work with the vacuum amplitude in Minkowski space rather than with Euclidean partition function to avoid problems with Majorana–Weyl spinors in Euclidean space.

The action (1.2) is invariant under the $\mathcal{N} = 2$ supersymmetry transformations

$$\begin{aligned} \delta^{(1)} \psi_\alpha^{ij} &= \frac{i}{2} [A_\mu, A_\nu]^{ij} (\Gamma^{\mu\nu} \epsilon)_\alpha, \\ \delta^{(1)} A_\mu^{ij} &= i\bar{\epsilon} \Gamma_\mu \psi^{ij}, \end{aligned} \quad (1.3)$$

and

$$\begin{aligned} \delta^{(2)} \psi_\alpha^{ij} &= \xi_\alpha \delta^{ij}, \\ \delta^{(2)} A_\mu^{ij} &= 0. \end{aligned} \quad (1.4)$$

The formulas look like as if ten-dimensional super Yang-Mills theory is reduced to a point¹.

The type IIB superstring theory consistently incorporates [10] Dp-branes with odd p ($p = -1, 1, 3, 5, \dots$). In order for the matrix model to describe nonperturbative dynamics of type IIB string, it should correctly reproduce the central charges in the supersymmetry algebra. These central charges have nontrivial tensor structure and are associated with D-branes of various dimensions.

¹Another matrix model on a point was advocated in [9].

The action (1.2) is, up to a constant term, the low energy effective action of D-instanton (associated with $p = -1$) of charge n [2]. Higher dimensional branes are expected to show up in the matrix model as solutions of the classical equations

$$[A^\mu [A_\mu, A_\nu]] = 0, \quad [A_\mu, (\Gamma^\mu \psi)_\alpha] = 0, \quad (1.5)$$

which are to be solved for $n \times n$ matrices A_μ at infinite n . A general solution has a block-diagonal form and is composed from non-diagonal $n_i \times n_i$ matrices with various n_i . A simplest solution corresponds to a diagonal matrix

$$A_\mu^{\text{cl}} = \text{diag} (p_\mu^{(1)}, \dots, p_\mu^{(n)}) , \quad \psi_\alpha = 0. \quad (1.6)$$

In analogy with Ref. [1], each of p_μ 's is to be identified with the coordinates of D-instanton which generates space-time coordinates.

As is discussed in [8], D-strings (associated with $p = 1$) are also described by the matrix model (1.1) using the idea of Ref. [1] to identify D-branes with operator-like solutions of Eq. (1.5). A static D-string extending along the $\mu = 1$ axis is represented by

$$A_\mu^{\text{cl}} = (B_0, B_1, 0, \dots, 0), \quad \psi_\alpha^{\text{cl}} = 0, \quad (1.7)$$

where the operators (infinite $n \times n$ matrices) B_0 and B_1 obey canonical commutation relation on a torus. The torus is associated with large compactification radii, T and L , along the $\mu = 0, 1$ directions so that the ratio $TL/n = \alpha'$ is kept fixed as $n \rightarrow \infty$. As is shown in [8], the interaction between two classical solutions (1.7), calculated at the one loop level in the matrix model (1.1), agrees with that of D-strings in IIB supergravity. This confirms the identification of the classical solution (1.7) with D-string. The emergence of the Born-Infeld action has been also discussed [11].

In the present paper we consider how three- and five-branes are described by the matrix model (1.1). In Sect. 2 we calculate central charges in supersymmetry algebra at infinite n and associate them with one- and five-branes present in IIB superstring theory. In Sect. 3 we consider classical solutions associated with three- and five-branes and calculate their interactions at one loop in the matrix model. In Sect. 4 we give a general prescription for taking the large n limit appropriate for the description of Dp-branes in the IKKT matrix model. Finally we discuss in Sect. 5 some aspects of the matrix-model formulation of IIB superstring.

2 Central charges in supersymmetry algebra

The supersymmetry transformations (1.3) and (1.4), under which the action (1.2) is invariant, are generated by two supercharges

$$Q_\alpha^{(1)} = -\frac{i}{2} [A_\mu, A_\nu]^{ij} \Gamma_{\alpha\beta}^{\mu\nu} \frac{\partial}{\partial \bar{\psi}_\beta^{ij}} + i \Gamma_{\mu\alpha\beta} \psi_\beta^{ij} \frac{\partial}{\partial A_\mu^{ij}} \quad (2.1)$$

and

$$Q_\alpha^{(2)} = \frac{\partial}{\partial \bar{\psi}_\alpha^{ji}}. \quad (2.2)$$

These operators form the $\mathcal{N} = 2$ supersymmetry algebra [8], which is not central extended at finite n .

The situation changes at $n = \infty$. As was shown in [6], the supersymmetry algebras in matrix models can acquire central charges in the infinite n limit. This happens because the quantities proportional to the traces of commutators, which vanish for finite matrices and are usually dropped in the calculation of the anticommutation relations in supersymmetry algebra, can be not equal to zero for operators in the Hilbert space. If matrix commutators are replaced in the large n limit by Poisson brackets and the traces are substituted by the integrals over parameter space, the trace of the commutator takes the form of an integral of the full derivative what is typical for central charges. Such terms should be retained and lead to the central extension of the supersymmetry algebra. In the BFSS matrix model they were calculated in [6]. We shall perform the analogous calculation for the IKKT matrix model.

We introduce the operators

$$P_{ij}^\mu = \frac{\partial}{\partial A_\mu^{ji}}, \quad (2.3)$$

$$\bar{\chi}_{\alpha ij} = \frac{\partial}{\partial \psi_\alpha^{ji}}, \quad \chi_{\alpha ij} = -\frac{\partial}{\partial \bar{\psi}_\alpha^{ji}}. \quad (2.4)$$

Note that ψ and χ have opposite chirality. We shall denote (anti)commutators of differential operators by $[\ ,]_\pm$, while for matrices we shall use the symbols $\{ \ , \}$ and $[\ ,]$. We follow the convention that matrix (anti)commutators do not change an operator ordering. For example,

$$[A, B]_{ij} \equiv A_{ik} B_{kj} - A_{kj} B_{ik}. \quad (2.5)$$

In this notations, the generator of the infinitesimal gauge transformation,

$$\begin{aligned} \delta_{\text{gauge}} A_\mu &= i [A_\mu, \Omega], \\ \delta_{\text{gauge}} \psi_\alpha &= i [\psi_\alpha, \Omega] \end{aligned} \quad (2.6)$$

reads

$$\Phi_{ij} = [A_\mu, P^\mu]_{ij} - [\bar{\psi}, \chi]_{ij}. \quad (2.7)$$

We define the operators

$$q_{\alpha ij}^{(1)} = \frac{i}{4} \{ [A_\mu, A_\nu], (\Gamma^{\mu\nu} \chi)_\alpha \}_{ij} + \frac{i}{2} \{ (\Gamma_\mu \psi)_\alpha, P^\mu \}_{ij} \quad (2.8)$$

and

$$q_{\alpha ij}^{(2)} = -\chi_{\alpha ij}, \quad (2.9)$$

which are the counterparts of supercharge densities in the BFSS matrix model, since

$$Q_\alpha^{(1),(2)} = \text{Tr } q_\alpha^{(1),(2)}. \quad (2.10)$$

To find the central charges in supersymmetry algebra, we first calculate the anticommutators of the densities and the supercharges. In this rather lengthy calculation we use the following Fierz identity for ten-dimensional Majorana–Weyl spinors:

$$\begin{aligned}
(\Gamma_\mu \psi)_{(\alpha} \otimes (\Gamma^{\mu\nu} \chi)_{\beta)} &= 2(\Gamma^\nu \Gamma^0)_{\alpha\beta} \bar{\psi} \otimes \chi - \frac{7}{8} (\Gamma^\mu \Gamma^0)_{\alpha\beta} \bar{\psi} \Gamma^\nu \Gamma_\mu \otimes \chi \\
&+ \frac{1}{8 \cdot 5!} (\Gamma^{\mu\lambda\rho\sigma\tau} \Gamma^0)_{\alpha\beta} \bar{\psi} \Gamma^\nu \Gamma_{\mu\lambda\rho\sigma\tau} \otimes \chi.
\end{aligned} \tag{2.11}$$

Using this formula we finally obtain

$$\begin{aligned}
[q_{\alpha ij}^{(2)}, Q_\beta^{(2)}]_+ &= 0, \\
[q_{\alpha ij}^{(1)}, Q_\beta^{(2)}]_+ &= -i(\Gamma_\mu \Gamma^0)_{\alpha\beta} P_{ij}^\mu, \\
[q_{\alpha ij}^{(1)}, Q_\beta^{(1)}]_+ &= 2(\Gamma^\mu \Gamma^0)_{\alpha\beta} z_{\mu ij} + 2(\Gamma^{\mu\nu\lambda\rho\sigma} \Gamma^0)_{\alpha\beta} z_{\mu\nu\lambda\rho\sigma ij} \\
&- 2(\Gamma^\mu \Gamma^0)_{\alpha\beta} \{A_\mu, \Phi\}_{ij} + \frac{7}{8} (\Gamma^\mu \Gamma^0)_{\alpha\beta} \{[\bar{\psi}, A_\nu] \Gamma^\nu \Gamma_\mu, \chi\}_{ij} \\
&- \frac{1}{8 \cdot 5!} (\Gamma^{\mu\lambda\rho\sigma\tau} \Gamma^0)_{\alpha\beta} \{[\bar{\psi}, A_\nu] \Gamma^\nu \Gamma_{\mu\lambda\rho\sigma\tau}, \chi\}_{ij},
\end{aligned} \tag{2.12}$$

where

$$\begin{aligned}
z_\mu &= [A_\nu, \{A_\mu, P^\nu\}] - [\bar{\psi}, A_\mu \chi] - [A_\mu \bar{\psi}, \chi] + \frac{7}{32} [\bar{\psi} A_\nu \Gamma^\nu \Gamma_\mu, \chi], \\
z_{\mu\lambda\rho\sigma\tau} &= -\frac{1}{32 \cdot 5!} [\bar{\psi} A_\nu \Gamma^\nu \Gamma_{\mu\lambda\rho\sigma\tau}, \chi].
\end{aligned} \tag{2.13}$$

Taking the trace of Eq. (2.12), we find that, up to the gauge transformations and equations of motion for $\bar{\psi}$, the supercharges obey the anticommutation relations

$$\begin{aligned}
[Q_\alpha^{(2)}, Q_\beta^{(2)}]_+ &= 0, \\
[Q_\alpha^{(1)}, Q_\beta^{(2)}]_+ &= -i(\Gamma_\mu \Gamma^0)_{\alpha\beta} \text{Tr } P^\mu, \\
[Q_\alpha^{(1)}, Q_\beta^{(1)}]_+ &= (\Gamma^\mu \Gamma^0)_{\alpha\beta} Z_\mu + (\Gamma^{\mu\nu\lambda\rho\sigma} \Gamma^0)_{\alpha\beta} Z_{\mu\nu\lambda\rho\sigma}.
\end{aligned} \tag{2.14}$$

The central charges, $Z_\mu = \text{Tr } z_\mu$ and $Z_{\mu\nu\lambda\rho\sigma} = \text{Tr } z_{\mu\nu\lambda\rho\sigma}$, being equal to the traces of the commutators, vanish for finite n . But at $n = \infty$ they are not necessarily turn to zero and we associate them with one- and five-branes present in type IIB superstring theory.

It is worth mentioning that all the charges are operator-valued and their interpretation is not as clear as for those of Ref. [6] in the BFSS matrix model, where the value of the charges is given by substituting the classical solution. Also there is no three-brane charge in the supersymmetry algebra. Similarly, the five-brane charge has purely fermionic nature. This circumstance may cause difficulties in the description of three- and five-branes as certain classical field configurations of the matrix model. Nevertheless, in the next section we shall study some classical solutions of the matrix model, which can be seemingly interpreted as D-branes of different dimensions.

3 Brane–brane interaction

It was argued in [6, 8] that for BPS states the field strength

$$f_{\mu\nu} = i[A_\mu, A_\nu], \quad (3.1)$$

should be proportional to the unit matrix. The classical equations (1.5) are in this case automatically satisfied. Since D-branes are BPS-states [10], classical solutions of the matrix model which correspond to D-branes should have this property.

Motivated by the four-brane solution found in [6] for the BFSS matrix model, we associate with a static Dp-brane the following classical solution of the model (1.2):

$$A_\mu^{\text{cl}} = (B_0, B_1, B_2, \dots, B_p, 0, \dots, 0), \quad \psi_\alpha^{\text{cl}} = 0, \quad (3.2)$$

where B_0, \dots, B_p are operators (infinite matrices) with the commutator

$$[B_a, B_b] = -ig_{ab}\mathbf{1}, \quad (3.3)$$

where $a, b = 0, \dots, p$. Since p is odd, these B_a 's can be written as linear combinations of $(p+1)/2$ pairs of canonical variables p_k, q_k ($k = 1, \dots, (p+1)/2$) satisfying $[q_k, p_l] = i\delta_{kl}$. The solution (3.2) is an obvious extension of (1.7). The property (3.3) guarantees that the effective action in the background (3.2) does not acquire quantum corrections.

The configuration containing a pair of Dp-branes can then be constructed embedding the classical solutions (3.2) in A_μ diagonally. We shall study in this section most general background configurations of this type, which are very similar to the one considered in the context of the BFSS matrix model in [7] and generalize the configurations with two static D-strings of Ref. [8].

The natural choice of the classical solution which can be interpreted as two parallel (antiparallel) Dp-branes at the distance b from each other is

$$\begin{aligned} A_a^{\text{cl}} &= \begin{pmatrix} B_a & 0 \\ 0 & B'_a \end{pmatrix}, \quad a = 0, \dots, p \\ A_{p+1}^{\text{cl}} &= \begin{pmatrix} b/2 & 0 \\ 0 & -b/2 \end{pmatrix}, \\ A_i^{\text{cl}} &= 0, \quad i = p+2, \dots, 9, \end{aligned} \quad (3.4)$$

where

$$[B_a, B_b] = -ig_{ab}\mathbf{1}, \quad [B'_a, B'_b] = -ig'_{ab}\mathbf{1}. \quad (3.5)$$

For this configuration

$$f_{ab} = \begin{pmatrix} g_{ab} & 0 \\ 0 & g'_{ab} \end{pmatrix} = d_{ab} \otimes \mathbf{1}_2 + c_{ab} \otimes \sigma^3, \quad (3.6)$$

$$d_{ab} = \frac{g_{ab} + g'_{ab}}{2}, \quad (3.7)$$

$$c_{ab} = \frac{g_{ab} - g'_{ab}}{2}, \quad (3.8)$$

and all other $f_{\mu\nu}$ are equal to zero. The matrix c_{ab} can always be brought to the canonical form by Lorentz transformation, so without loss of generality we can assume that

$$c_{ab} = \begin{pmatrix} 0 & -\omega_1 & & & \\ \omega_1 & 0 & & & \\ & & \ddots & & \\ & & & 0 & -\omega_{\frac{p+1}{2}} \\ & & & \omega_{\frac{p+1}{2}} & 0 \end{pmatrix}. \quad (3.9)$$

If the Dp-branes are parallel, we can set $B_a = B'_a$ by a canonical transformation, so that $c_{ab} = 0$. This corresponds again to the BPS-saturated case. If the Dp-branes are antiparallel, say, along one axis, then $c_{ab} \neq 0$ and their interaction is to be calculated. We use for this purpose the result of Ref. [8] for the one-loop effective action around a general background A_μ^{cl} satisfying Eq. (1.5):

$$W = \frac{1}{2} \text{Tr} \ln(P^2 \delta_{\mu\nu} - 2iF_{\mu\nu}) - \frac{1}{4} \text{Tr} \ln \left((P^2 + \frac{i}{2} F_{\mu\nu} \Gamma^{\mu\nu}) \left(\frac{1 + \Gamma_{11}}{2} \right) \right) - \text{Tr} \ln(P^2), \quad (3.10)$$

where the adjoint operators P_μ and $F_{\mu\nu}$ are defined on the space of matrices by

$$P_\mu = [A_\mu^{\text{cl}}, \cdot], \quad F_{\mu\nu} = [f_{\mu\nu}^{\text{cl}}, \cdot] = i [[A_\mu^{\text{cl}}, A_\nu^{\text{cl}}], \cdot]. \quad (3.11)$$

$\text{Im } W$ vanishes for $p = 1, 3, 5, 7$ since we have $P_\mu = 0$ at least in one direction.

The calculation of (3.10) considerably simplifies for the background (3.4) when all of the operators P_μ and $F_{\mu\nu}$ have the form $\mathcal{O}_1 \otimes 1 + \mathcal{O}_3 \otimes \Sigma^3$ with $\Sigma^3 = [1 \otimes \sigma^3, \cdot]$. Thus they commute with Σ^3 and the eigenfunctions of the operators entering (3.10) can be classified according to the eigenvalues of Σ^3 . The terms corresponding to zero eigenvalues of Σ^3 do not contribute to the effective action (3.10). Two other eigenvalues of Σ^3 are ± 2 and they give equal contributions. The commutation relations

$$F_{ab} = i[P_a, P_b] = c_{ab} \Sigma^3 \quad (3.12)$$

and the equality

$$P_{p+1} = \frac{b}{2} \Sigma^3 \quad (3.13)$$

show that after analytical continuation to the Euclidean space the operator P^2 projected on the eigenspace of Σ_3 can be regarded as a Hamiltonian of $\left(\frac{p+1}{2}\right)$ -dimensional harmonic oscillator with frequencies ω_i . Its eigenvalues thus are

$$E_{\mathbf{k}} = 4 \sum_{i=1}^{\frac{p+1}{2}} \omega_i \left(k_i + \frac{1}{2} \right) + b^2. \quad (3.14)$$

Each of them has n -fold degeneracy.

For c_{ab} given by Eq. (3.9), the effective action can be brought, as was shown in [8], to the form

$$W = n \sum_{\mathbf{k}} \left[\sum_i \ln \left(1 - \frac{16\omega_i^2}{E_{\mathbf{k}}^2} \right) - \frac{1}{2} \sum_{\substack{s_1, \dots, s_5 = \pm 1 \\ s_1 \dots s_5 = 1}} \ln \left(1 - \frac{2 \sum_i \omega_i s_i}{E_{\mathbf{k}}} \right) \right]. \quad (3.15)$$

The sums over k_i can be calculated using the formulas

$$\ln \frac{u}{v} = \int_0^\infty \frac{dx}{x} (e^{-vx} - e^{-ux}) \quad (3.16)$$

and

$$\sum_{\mathbf{k}} e^{-x E_{\mathbf{k}}} = \frac{e^{-b^2 x}}{\prod_i 2 \sinh 2\omega_i x}. \quad (3.17)$$

After some algebra we get

$$W = -2n \int_0^\infty \frac{dx}{x} e^{-b^2 x} \left[\sum_i (\cosh 4\omega_i x - 1) - 4 \left(\prod_i \cosh 2\omega_i x - 1 \right) \right] \prod_i \frac{1}{2 \sinh 2\omega_i x}. \quad (3.18)$$

The integral is convergent for $p \leq 5$ and logarithmically divergent for $p = 7$. Retaining only the leading term in $1/b^2$, we obtain from Eq. (3.18):

$$\begin{aligned} W &= -\frac{1}{16} n \left(\frac{5-p}{2} \right)! \left[2 \sum_i \omega_i^4 - \left(\sum_i \omega_i^2 \right)^2 \right] \prod_i \omega_i^{-1} \left(\frac{2}{b} \right)^{7-p} + O \left(\frac{1}{b^{9-p}} \right) \\ &= -\frac{1}{64} n \left(\frac{5-p}{2} \right)! \left[4c_{ab}c_{bd}c_{de}c_{ea} - (c_{ab}c_{ba})^2 \right] \left(\det_{ab} c_{ab} \right)^{-1/2} \left(\frac{2}{b} \right)^{7-p} \\ &\quad + O \left(\frac{1}{b^{9-p}} \right). \end{aligned} \quad (3.19)$$

The right-hand side of Eq. (3.19) obviously vanishes for parallel Dp-branes when $c_{ab} = 0$ and recovers the result of Ref. [8] for $p = 1$. For $p = 3, 5$ it gives a consistent result for the interaction between two antiparallel Dp-branes which falls as $1/b^{7-p}$ at large distances, as expected.

It is worth mentioning that for $p = 3$ and all ω_i 's equal to each other the coefficient in (3.19) turns to zero. Moreover, the complete effective action (3.18) vanishes in this case. However, it does not mean that antiparallel tree-branes do not interact, because the choice of equal ω_i is not appropriate for studying the interaction between branes. As is the case of D-strings [8], it is natural to put

$$\omega_1 = \frac{TL_1}{2\pi n^{\frac{2}{p+1}}} \quad (3.20)$$

where L_1 is large compactification radius in x_1 direction and T is the interval of time periodicity. Analogously, it is natural to set

$$\omega_i = \frac{L_{2i-2}L_{2i-1}}{2\pi n^{\frac{2}{p+1}}}. \quad (3.21)$$

The effective action (3.18) is related to the interaction potential when $T \gg L_a$. In this case the coefficient before b^{p-7} is always negative, thus the antiparallel branes always attract, as they should. The fact that the effective action vanishes for coinciding radii of compactification, although having nothing to do with the interaction between D-branes, may have sense, and it would be interesting to find a simple explanation of it.

The correct large distance behaviour of the interaction potential confirms the conjecture to identify the solution (3.4) with Dp-brane configurations. However, since they are not straightforwardly associated, as is already noted, with the central charges calculated in the previous section, other checks of this conjecture, in particular the derivation of the Born–Infeld action, would be useful.

4 The large n limit for D-branes

In this section we shall give a prescription for taking the large n limit appropriate for the description of D-branes in the IKKT matrix model of IIB string theory. Analogous prescription in the BFSS matrix model was given in [6]. Using this prescription, we obtain the correct dependence of physical quantities, such as the effective action for brane–brane system, on the fundamental constants.

The action (1.2) of the IKKT matrix model is actually the action for the $p = -1$ D-brane (D-instanton), so the higher dimensional D-branes can be viewed as the composites of instantons similarly to the BFSS matrix model, in which D-branes are composed from D0-branes. In [1] it was shown that the transverse density of partons (D0-branes) is strictly bounded to about one per transverse Planck area. In other words the partons form a kind of incompressible fluid. We assume that an analogous property holds for the IKKT matrix model.

Let V_{p+1} be a large enough volume of the world-volume of the p-brane. We choose n to be

$$n \sim \frac{V_{p+1}}{l_s^{p+1}}, \quad (4.1)$$

where $l_s = \sqrt{\alpha'}$ is the string length scale. The physical picture of this is that the p-brane world-volume is constituted of n cells of volume l_s^{p+1} . This choice of n turns out to give correct dependence of physical quantities on the characteristic constants.

For A_μ^{cl} to have a dimension of length, the constants g_{ab} in eq. (3.3) should be proportional to $V_{p+1}^{\frac{2}{p+1}}$ and should scale with n as $n^{-\frac{2}{p+1}}$ according to the arguments of [6] which

are based on the fact that the full Hilbert space of the dimension n is represented as the tensor product of $(p+1)/2$ Hilbert spaces of the dimension $n^{\frac{2}{p+1}}$ each. As a result, we get

$$[A_\mu, A_\nu] \sim V_{p+1}^{\frac{2}{p+1}} n^{-\frac{2}{p+1}} \quad (4.2)$$

for the commutator.

The bosonic part of the D-instanton action is

$$S \sim \frac{1}{g_s l_s^4} \text{Tr} [A_\mu, A_\nu]^2, \quad (4.3)$$

where g_s is the string coupling constant. Now, the substitution of Eq. (4.2) into Eq. (4.3) gives

$$S \sim \frac{1}{g_s l_s^4} V_{p+1}^{\frac{4}{p+1}} n^{\frac{p-3}{p+1}}. \quad (4.4)$$

Substituting our choice Eq. (4.1) for n into Eq. (4.4), we get

$$S \sim \frac{1}{g_s l_s^{p+1}} V_{p+1} \sim T_p V_{p+1}, \quad (4.5)$$

i.e. the action of p-brane = tension \times volume of the world-volume.

In the previous section we have computed the effective action for the configuration of two antiparallel D-branes. Taking into account the scaling law (4.2) which yields for c_{ab} :

$$c \sim V_{p+1}^{\frac{2}{p+1}} n^{-\frac{2}{p+1}}, \quad (4.6)$$

we find from eq. (3.19) that

$$W \sim n c^{\frac{7-p}{2}} b^{p-7} \sim V_{p+1} l_s^{6-2p} b^{p-7}. \quad (4.7)$$

This agrees with the known result from the theory of D-branes [10].

5 Discussion

Most of the checks, done so far, of the proposal that the IKKT matrix model is a nonperturbative formulation of IIB superstring deal with description of D-branes. Our paper is also along this line.

On the other hand, the matrix model (1.1) should reproduce string perturbation theory as well. As was argued in [8], that if large values of n and smooth matrices A_μ^{ij} and ψ_α^{ij} dominate in (1.1), the commutator can be substituted by the Poisson bracket

$$[\cdot, \cdot] \implies i\{\cdot, \cdot\} \quad (5.1)$$

and the trace can be substituted by the integration over parameters $\sigma = (\sigma_1, \sigma_2)$:

$$\text{Tr} \dots \implies \int d^2 \sigma \sqrt{|g(\sigma)|} \dots \quad (5.2)$$

so that the sum over n and matrix integrals in (1.1) turn into path integrals over a positive definite function $\sqrt{|g(\boldsymbol{\sigma})|}$ and over $X^\mu(\boldsymbol{\sigma})$ and $\psi_\alpha(\boldsymbol{\sigma})$:

$$\mathcal{Z} = \int D\sqrt{|g|} DX D\psi e^{i\mathcal{S}}, \quad (5.3)$$

while Eqs. (1.2)–(1.4) turn into

$$\mathcal{S} = \int d^2\sigma \left(\sqrt{|g|} \frac{1}{g_s(\alpha')^2} \left(-\frac{1}{4} \{X^\mu, X^\nu\}^2 + \frac{i}{2} \bar{\psi} \Gamma^\mu \{X_\mu, \psi\} \right) + \beta \sqrt{|g|} \right), \quad (5.4)$$

$$\begin{aligned} \delta^{(1)}\psi_\alpha &= -\frac{1}{2}\sqrt{|g|}\{X_\mu, X_\nu\}(\Gamma^{\mu\nu}\epsilon)_\alpha, \\ \delta^{(1)}X^\mu &= i\bar{\epsilon}\Gamma^\mu\psi, \end{aligned} \quad (5.5)$$

and

$$\begin{aligned} \delta^{(2)}\psi_\alpha &= \xi_\alpha, \\ \delta^{(2)}X^\mu &= 0. \end{aligned} \quad (5.6)$$

Here the Poisson bracket is defined by

$$\{X, Y\} \equiv \frac{1}{\sqrt{|g|}} \varepsilon^{ab} \partial_a X \partial_b Y. \quad (5.7)$$

The parameters ϵ_α and ξ_α do not depend on σ_1 and σ_2 similar to these in Eqs. (1.3) and (1.4) which are numbers rather than matrices.

This transition from matrices to functions of $\boldsymbol{\sigma}$ can be formalized introducing the matrix function

$$L(\boldsymbol{\sigma})^{ij} = \sum_{\mathbf{m}} j_{\mathbf{m}}(\boldsymbol{\sigma}) J_{\mathbf{m}}^{ij}, \quad (5.8)$$

where the index $\mathbf{m} = (m_1, m_2) \in \mathbf{Z}^2$, while $J_{\mathbf{m}}^{ij}$ form a basis for \mathfrak{gl}_∞ and $j_{\mathbf{m}}(\boldsymbol{\sigma})$ form a basis in the space of functions of $\boldsymbol{\sigma}$. An explicit form of $j_{\mathbf{m}}(\boldsymbol{\sigma})$'s depends on the topology of the $\boldsymbol{\sigma}$ -space. Explicit formulas are available for a sphere and a torus.²

The commutators of $J_{\mathbf{m}}$'s coincide with the Poisson brackets of $j_{\mathbf{m}}$'s at least for finite \mathbf{m} 's. This demonstrates the equivalence between the group of area-preserving or symplectic diffeomorphisms (Sdiff) and the gauge group $\text{SU}(\infty)$ for smooth configurations.

With the aid of (5.8) we can relate matrices with functions of $\boldsymbol{\sigma}$ by

$$A_\mu = \int d^2\boldsymbol{\sigma} \sqrt{|g|} X_\mu(\boldsymbol{\sigma}) L(\boldsymbol{\sigma}) \quad (5.9)$$

and vice versa

$$X_\mu(\boldsymbol{\sigma}) = \text{Tr } A_\mu L(\boldsymbol{\sigma}), \quad (5.10)$$

²For a review of this subject see [12] and references therein.

where the consequence of the completeness condition,

$$\text{Tr } L(\boldsymbol{\sigma})L(\boldsymbol{\sigma}') = \frac{1}{\sqrt{|g|}} \delta^{(2)}(\boldsymbol{\sigma} - \boldsymbol{\sigma}'), \quad (5.11)$$

has been used. The above formulas lead for smooth configurations to Eqs. (5.1) and (5.2). The word “smooth” here and above means precisely that configurations can be reduced by a gauge transformation to the form when high modes are not essential in the expansions (5.9) or (5.10).

Equations (5.3) and (5.4) represent IIB superstring in the Schild formalism with fixed κ -symmetry [8]. At fixed $\sqrt{|g|}$ the action (5.4) is invariant only under symplectic diffeomorphisms

$$\delta X_\mu = -\{X_\mu, \Omega\}, \quad \delta \psi_\alpha = -\{\psi_\alpha, \Omega\}, \quad (5.12)$$

(in the infinitesimal form). This is an analogue of the gauge transformation (2.6) in the matrix model (1.1) at fixed n which itself plays the role of $\sqrt{|g(\boldsymbol{\sigma})|}$. The full reparametrization invariance of the string is restored when one integrates over $\sqrt{|g(\boldsymbol{\sigma})|}$, which is an analog of the summation over n in (1.1). The matrix-model formulation is extremely nice from the point of view of fixing the symmetry under symplectic diffeomorphisms since this can be done by a standard procedure of fixing the gauge in gauge theory. It is that made it possible to calculate brane-brane interaction by doing the one-loop calculation in the IKKT matrix model.

A question arises whether or not these two procedures of fixing the symmetry would always give the same result or, in other words, are these two groups equivalent at the quantum level. An answer to this question depends on what configurations are essential in quantum fluctuations, and hence what modes are essential in the expansions (5.9) and (5.10). The answer to this question is known for a pure bosonic string where configurations which are not smooth are certainly important (at least in Euclidean space). They result in crumpled surfaces associated with tachionic excitations. Since there is no tachion for superstring at least perturbatively, one might expect that only smooth configurations are important in this case.

Another point of interest in superstring theory is calculation in perturbation theory where higher orders in the string coupling constant g_s are associated with non-trivial topologies of the parameter space. The string perturbation theory should presumably arise as a result of the loop expansion of the matrix model. In the above language of the relations (5.9) and (5.10) the higher terms of string perturbation theory could be perhaps associated with a non-trivial choice of the basis functions $j_{\mathbf{m}}$ ’s corresponding to a given topology. The algebra of symplectic diffeomorphism for non-trivial topologies was studied and, in particular, the presence of central charges was discovered for torus [13] and higher genera [14]. Analogously, it was discussed that the large N limit of $SU(N)$ is not unique [15, 16] and central extensions are possible. This fact might be of interest for investigations of the matrix model.

The central point in the IKKT approach is the presence of the β in (1.2) (and correspondingly in (5.4)). As is well known, Schield strings are tensionless for $\beta = 0$, and the string tension is proportional to $\sqrt{\beta}$.

A very interesting idea of Ref. [8] is that $\beta \neq 0$ can appear dynamically in the Eguchi–Kawai reduced ten-dimensional super Yang–Mills theory specified by³

$$Z_{\text{EK}} = \int d\mathcal{A} d\Psi e^{iS} \quad (5.13)$$

with the action

$$S_{\text{EK}} = \frac{Na^4}{g_0^2} \left(\frac{1}{4} \text{Tr} [\mathcal{A}_\mu, \mathcal{A}_\nu]^2 + \frac{1}{2} \text{Tr} (\bar{\Psi} \Gamma^\mu [\mathcal{A}_\mu, \Psi]) \right) \quad (5.14)$$

where the $N \times N$ matrices \mathcal{A}_μ and Ψ_α have, respectively, the dimension of [mass] and [mass]^{3/2}, g_0^2 is dimensionless and a is a cutoff. In addition to the gauge symmetry (2.6), the model possesses the symmetry

$$\delta \mathcal{A}_\mu = \alpha_\mu \mathbf{1}_N \quad (5.15)$$

whose ten parameters α_μ depend on direction. This symmetry is crucial for vanishing of the averages of the type

$$\left\langle \frac{1}{N} \text{Tr} \mathcal{A}_\mu \right\rangle \equiv Z_{\text{EK}}^{-1} \int d\mathcal{A} d\Psi e^{iS} \frac{1}{N} \text{Tr} \mathcal{A}_\mu = 0 \quad (5.16)$$

with the integrand being invariant under the $\text{SU}(N)$ gauge transformation (2.6) but not under (5.15). The symmetry (5.15) is unbroken in the perturbation theory due to supersymmetry⁴ so that there is no need of quenching or twisting in contrast to large N QCD.

A mechanism of how the string perturbation theory could emerge in the reduced matrix models was discussed by Bars [19]. It is based on the $1/N$ expansion of the reduced model which leads, as for any matrix model, to the topological expansion according to general arguments by 't Hooft [20]. In order for contributions of higher genera not to be suppressed at large N , a kind of the double scaling limit is needed, which assumes usually fine tuning of the parameters. It would be very interesting to find out whether or not this mechanism works for the reduced model (5.13) and whether or not the double scaling procedure suggested in [8] could provide this.

It was proposed in [8], that the term with β in (1.2) which is not present in (5.14) can be generated in the reduced model within loop expansion. The problem of constructing loop expansion in the reduced model around plane vacuum, given by the classical solution (1.6), resides in zero modes of the fermionic matrix which exist due to the supersymmetry (1.4). It is still an open problem to show how the integral over the fermionic zero modes becomes nonvanishing.

³For a review of the reduced models see [17] and references therein.

⁴This fact was first advocated in [18] for four dimensions.

We would like to speculate on a potential way to deal with this problem which is related to the fact that the integral over bosonic zero modes

$$\int \prod_{i=1}^N d^{10} p_{\mu}^{(i)} = \infty \quad (5.17)$$

is formally divergent if p_{μ} 's are not quenched. Therefore, an uncertainty of the type $\infty \cdot 0$ appears in nonregularized theory which is to be done. A useful hint on how the result can look like is given by the simplified model [21] where the partition function was calculated via the Nicolai map.

There exists one more potential way out for the problem of the fermionic zero modes — the same as in superstring theory — where simplest amplitudes are well-known to vanish exactly for the same reason. One should consider in the reduced model averages of several operators (analogous to vertex operators in superstring theory) to make the integral over the fermionic zero modes nonvanishing.

Acknowledgments

We thank A. Gorsky, A. Mikhailov, A. Morozov and P. Olesen for useful discussions. This work was supported in part by INTAS grant 94–0840 and by CRDF grant 96–RP1–253. Y.M. was sponsored in part by the Danish Natural Science Research Council. The work of K.Z. was supported in part by RFFI grant 96–01-00344.

Note added

After this paper was submitted for publication, the revised version of [22] has appeared which proposes to interpret the classical solutions in the IKKT matrix model as bound states of D-branes with large (of order n) number of D-instantons, in analogy with previous work [5] on the BFSS matrix model.

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